

Referee's Report on
Pan-connectedness of graphs with large neighborhood
unions (GC2274)
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A graph G with n vertices is said to be $[m, n]$ -panconnected, if for any pair x, y of vertices of G there is an (x, y) -path in G of every length l for $m \leq l \leq n$. The pan-connectedness is a very strong property for graphs. In this paper, the authors give a sufficient condition for a 3-connected graph to be $[5, n]$ -panconnected. This condition is based on the sum of the minimum cardinality of the neighborhood unions among all pairs of vertices at distance two and the minimum degree of the graph. As a consequence, a conjecture proposed by Wei and Zhu in 1998 is confirmed. Under the same condition, the authors also characterize all classes of 2-connected graphs which are not $[4, n]$ -pan-connected.

I think the main results of the paper are fairly interesting and new. But the paper is not well prepared. There are some typos and a gap in the proof of Lemma 4.3, which need to be fixed. I want to see the revised paper. The following are my detailed comments.

Comments

Throughout the paper the authors use the notations " \forall " and " \exists " which are not appeared very often in the published papers in Graphs and Combinatorics. I suggest to replace them by "for all" or "for any" and "there exist" or "there exists", respectively.

Page 1

Line 3 in Abstract, replace "has" by "are at".

Page 2

Line 9, replace "have been obtained" by "are obtained".

Line 15, replace " $[5, n]$ -connected" by " $[5, n]$ -pan-connected".

Line -3, replace " $\{G_1, G_2, G_3\}$ " by " $\{G_1, G_2, G_3, G_4\}$ " and replace "Figure 1.1–1.3" by "Figures 1.1–1.4".

Page 3

Line 2, replace "Figure 14: G_1 " by "Figure 14: G_4 ".

Line 11, give an explanation for the graph " $L_1 \cup (L_2 \vee L_3)$ " and add the edge " x_0y_2 ".

Page 4

Line 2, add "the" before "following".

Line 12, replace the second " $=$ " by " \leq ".

Line -5, deleted one "and".

Page 5

Lines 2 and 3, replace "length" by "lengths".

Line 15, replace " $N(x_1) - \{x_0\} \cup \{u\}$ " by " $(N(x_1) - \{x_0\}) \cup \{u\}$ ".

Line 19, replace " $N(u) - \{x_0\} \cup \{x_2\}$ " by " $(N(u) - \{x_0\}) \cup \{x_2\}$ ".

Line -4, replace " $N[x_1] - \{x_0, x_2\} \cup \{u_1\}$ " by " $(N[x_1] - \{x_0, x_2\}) \cup \{u_1\}$ ".

Remark. This kind of typos appeared also many times in other places on pages 9 to 14, the authors should make the correspondent corrections.

Page 6

Lines 3–4, delete " an " and replace " path of length " by " paths of lengths ". The same applies for Lines 19 and 23.

Page 7

More work need to be done for the proofs of Lemma 4.3, since Corollary 4.2 can not be applied for P_{m+1} !

Page 8

Line -13, I can not find " Lemma 7 " in the paper!

Line -11, delete " Then $d(y_0, y_2) = 2$ ".

Page 9

Line 13, " $N_{G-P_2-\{y_1\}}$ " is not consisten with the other notations.

Line 17, replace " $N(x_2) - \{y_0^1\} \cup z_0$ " by " $(N(x_2) - \{y_0^1\}) \cup \{z_0\}$ ".

Page 10

First show $x_0x_2 \in E(G)$ by moving the paragraph start with " If $x_0x_2 \notin E(G)$, " befroe the paragraph start with " First we show....".

Page 11

Line 8, delete " that " after " contradiction ".

Line 10, replace " Case 1 assumption " by " the assumption of Case 1 ".

Line 14, add " which " before " are ".

Lines -18 and -14, replace " (x_i, x_{m-1}) path " by " (x_i, x_{m-1}) -path ", respectively.

Line -1, replace " as " by " to ".

Page 14

Line 2, replace " Fact " by " Facts ".

Line 15, replace " Subcase " by " Subcases ".

Line 16, replace " Theorem " by " Theorems ".

Line 18, replace " structure " by " Structures ".